

GIC Influence on Power Systems calculated by Carson's method

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Keywords: GIC, Electrojet, Carson, EMTP, Spreadsheet

ABSTRACT

During solar storms the ejection of an enormous amount of charged particles has been observed and the latter have created havoc on electrical systems. In this paper the electromagnetic influence of ionized Solar Mass Ejection on the conducting earth is equivalenced with the influence of an electrojet in the ionosphere coupled with the earth and the power lines. This coupling is computed by means of Carson's equations. The paper first summarizes the origin of these equations and their applicability, then uses them to make an approximation of the induced voltages and the magnetic induction at ground level relying on the use of the Carson ground return and terminates by comparing the same quantities using impedance matrices produced as output of an electromagnetic transients program.

RÉSUMÉ

Au cours de tempêtes solaires d'énormes masses de particules chargées sont éjectées dans l'espace et ont créé de sérieux problèmes dans les réseaux. Dans cet article l'influence de l'éjection de masse solaire ionisée sur la terre est représentée par le couplage d'un courant intense (électrojet) dans l'ionosphère avec un sol conducteur et avec les lignes à haute tension. Ce couplage est étudié en utilisant les formules de Carson. Dans un premier temps on rappelle l'origine de ces équations et leur domaine d'application pour ensuite en utiliser des approximations obtenues par développement en séries tronquées pour un calcul des tensions induites et de l'induction magnétique au niveau du sol. Ces résultats sont comparés à ceux obtenus à partir des matrices d'impédances produites par un programme pour le calcul de transitoires électromagnétiques.

SAMENVATTING

Tijdens zonnestormen worden enorme massas geladen deeltjes de ruimte ingeslingerd en die blijken de oorzaak te zijn geweest van zware incidenten in hoogspanningsnetten. In dit artikel wordt de invloed van de uitstoot van geïoniseerde zonnemassa op de aarde voorgesteld door de koppeling van een intense stroom in de ionosfeer (electrojet) met een geleidende aarde en hoogspanningslijnen. Deze koppeling wordt bestudeerd door gebruik te maken van de formules van Carson. In een eerste deel wordt de oorsprong van deze vergelijkingen en hun toepassingsgebied opgefrist om dan in een tweede deel de spanningen en de magnetische inductie op grondniveau door afgebroken reeksen te bepalen. Tenslotte wordt een vergelijking gemaakt met de resultaten bekomen door gebruik te maken van de impedantiematrixen geproduceerd als output van een elektromagnetische transiënten programma.

Introduction

The observation of the sun during solar storms has shown that Solar Mass Ejection (SME) expulse enough charged particles to perturbate the magnetic field at the surface of the earth and generate high current streams or layers in the ionosphere. The latter are called auroral electrojets and are related to the aurora borealis and aurora australis phenomena observed in the respective Arctic and Antarctic regions. These electrojets generate slowly varying near-DC electric fields and corresponding Geomagnetically Induced Currents (GIC) in underlying conductive grounds and high voltage lines. As important transformers are usually grounded, these near-DC homopolar currents may saturate the core and may therefore lead to overheating of the apparatus, excessive reactive power demand and power system instability.

The calculation of the electric field and the magnetic induction at ground level induced by an electrojet is the subject of this paper and relies on the use of Carson's

formulas, originally derived for studying the influence of finite conductivity of the ground on the impedance of horizontal long-wave radio antennas and their influence on parallel wiring. The resulting formulas have been applied in power line calculations for determining the homopolar line impedances and for evaluating the voltages and the currents induced in circuits parallel to power lines like telephone circuits and railways, but will now be investigated outside their usual domain of application.

The Carson model: basic approach and approximate formulas

In order to take into account the finite conductivity of the earth, J. R. Carson introduced correction terms to the self impedance of a conductor and to the mutual impedance between parallel conductors drawn above the earth when a sinusoidal excitation with angular frequency ω (rad/s) is applied [1].

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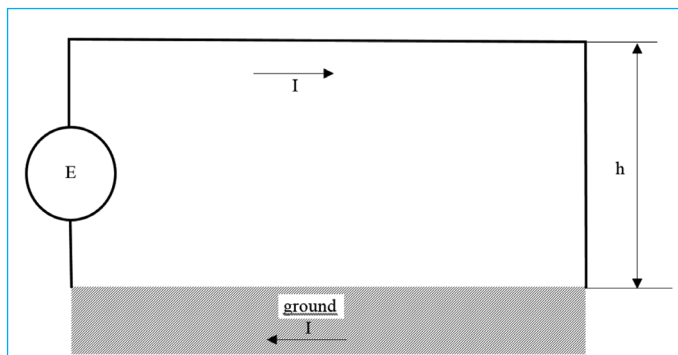


Fig. 1: Idealized conductor above ground return

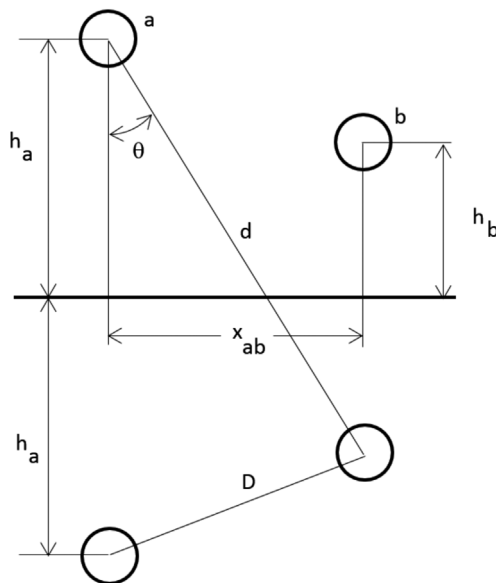


Fig. 2: Geometry of conductors a and b with their respective images

The main assumptions for the earth are:

- it is comprised in a half space limited only by an infinite plane, the ground surface,
- it has a uniform conductivity and a relative permeability equal to 1,
- the current in the ground runs parallel to the conductor(s). No end effects are considered.

Self-impedance

Consider first a single conductor with radius r running above and parallel to a perfectly conducting earth at a height h (Fig. 1). The resistance of the loop is therefore only the resistance of the conductor, given by:

$$R_{aa-g(\rho_a=0)} = \frac{\rho}{\pi r^2} \quad \Omega/\text{m}$$

ρ being the resistivity of the conductor having a radius r and subscripts having the following meaning:

aa indicates that the resistance relates to the conductor a due to current in a ,
 g indicates that the resistance relates to the resistance of the loop through the ground,
 $(\rho_a = 0)$ indicates that the resistivity of the ground is considered zero.

As the ground is supposed to be perfectly conducting and infinite, the flux generated by a current in the conductor cannot penetrate the ground and there is no possible flux linking the ground return. The inductance of the loop is thus limited to the inductance of the sole conductor, which equals half of the inductance of a loop consisting of the conductor and its image with respect to the ground surface. Therefore the reactance per unit length of the loop is given by:

$$X_{aa-g(\rho_a=0)} = \frac{1}{2} \cdot 4 \cdot 10^{-7} \cdot \omega \cdot \ln \frac{2h}{r'} \quad \text{H / m}$$

in which “ln” indicates the natural logarithm and r' is the so-called Geometric Mean Radius (GMR) and takes into account the internal flux linkage of the conductor. Because the frequencies considered in the GIC studies are extremely low, the GMR of the conductor may be set equal to $0,7788 \cdot r$.

If we consider now that the ground no longer is perfectly conducting, but exhibits a certain resistivity, the subscript will no longer carry $(\rho_a = 0)$, and the impedance of the ground loop is then written as:

$$Z_{aa-g} = (R_{aa-g(\rho_a=0)} + j \cdot X_{aa-g(\rho_a=0)}) + (\Delta R_{aa-g} + j \cdot \Delta X_{aa-g})$$

ΔR_{aa-g} and ΔX_{aa-g} are corrections due to the finite resistivity of the ground ($\rho_a \neq 0$)

Carson [1] has derived exact integral expressions for these corrections:

$$\Delta R_{aa-g} + j \Delta X_{aa-g} = 4 \cdot 10^{-7} \cdot \omega \cdot \int_0^{\infty} (\sqrt{\lambda^2 + j} - \lambda) \cdot \exp(-k\lambda) \cdot d\lambda$$

where λ is an integration variable and $k = 2h\sqrt{\omega\mu/\rho_a}$

with ρ_a = soil resistivity in Ωm and μ the permeability equal to $4\pi \cdot 10^{-7} \text{ H m}^{-1}$.

The value of these corrections is obtained by evaluating the integral by infinite series for the real and the imaginary part. Carson called them respectively P and Q :

$$\Delta R_{aa-g} + j \Delta X_{aa-g} = 4 \cdot 10^{-7} \omega (P_{\theta=0} + j Q_{\theta=0})$$

In this expression the subscript θ appears and its significance will be explained later on, as it relates only to the mutual impedance. For the self impedance θ can be put equal to zero.

For the important range where $k < 1$ an expression for P and Q is given in Carson's original paper. After converting them to SI units they are given by the following equations:

$$P_{\theta=0} = \frac{\pi}{8} - \frac{k}{3\sqrt{2}} + \frac{k^2}{16} (0,6728 + \ln \frac{2}{k}) + \frac{k^3}{45\sqrt{2}} - \frac{\pi k^4}{1536}$$

$$Q_{\theta=0} = -0,0386 + \frac{1}{2} \ln \frac{2}{k} + \frac{k}{3\sqrt{2}} - \frac{\pi k^2}{64} + \frac{k^3}{45\sqrt{2}} - \frac{k^4}{384} (\ln \frac{2}{k} + 1,0895)$$

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in which “ln” indicates the natural logarithm and k was defined above.

For values of $k > 5$ another approximating series for $P_\theta = 0$ and $Q_\theta = 0$ must be used:

$$P_{\theta=0} = \frac{1}{\sqrt{2} \cdot k} - \frac{1}{k^2} + \frac{1}{\sqrt{2} \cdot k^3} + 3 \frac{1}{\sqrt{2} \cdot k^5} \dots$$

and

$$Q_{\theta=0} = \frac{1}{\sqrt{2} \cdot k} - \frac{1}{\sqrt{2} \cdot k^3} + 3 \frac{1}{\sqrt{2} \cdot k^5} \dots$$

Mutual impedance

Consider two parallel conductors a and b at a height of h_a and h_b meter above ground (Fig. 2). Carson introduced the angle θ in his study of the mutual influence and this angle is clearly defined as the angle between the vertical line to the image of conductor a and the line to the image of conductor b .

If the ground were perfectly conducting the mutual impedance would be restricted to the mutual (imaginary) reactance, but if the ground has a finite conductivity a real term will appear. The general expression for the mutual impedance will therefore be:

$$Z_{ba-g} = j \cdot X_{ba-g(\rho_a=0)} + \Delta R_{ba-g} + j \Delta X_{ba-g}$$

where

$$X_{ba-g(\rho_a=0)} = 2 \cdot 10^{-7} \omega \ln \frac{d}{D}$$

The subscripts 'ba' indicate now that the impedance relates to the voltage induced in conductor b due to a current in conductor a . Of course, in this configuration there is reciprocity so that $X_{ab} = X_{ba}$

The exact correction terms were again derived by Carson in the form of an integral from zero to infinity of a complex function. Carson also derived an approximation of this integral for $k < 1$, where k has a slightly different meaning from the previous paragraph:

$$\Delta R_{ba-g} + j \Delta X_{ba-g} = 4 \cdot 10^{-7} \omega (P_\theta + j Q_\theta)$$

where

$$P_\theta = \frac{\pi}{8} - \frac{k \cos \theta}{3\sqrt{2}} + \frac{k^2 \cdot \cos 2\theta}{16} (0,6728 + \ln \frac{2}{k}) + \frac{k^2 \cdot \theta}{16} \cdot \sin 2\theta + \frac{k^3 \cdot \cos 3\theta}{45\sqrt{2}} - \frac{\pi k^4 \cdot \cos 4\theta}{1536} \quad (1)$$

$$Q_\theta = -0,03861 + \frac{1}{2} \ln \frac{2}{k} + \frac{k \cdot \cos \theta}{3\sqrt{2}} - \frac{\pi k^2 \cdot \cos 2\theta}{64} + \frac{k^3 \cos 3\theta}{45\sqrt{2}} - \frac{k^4 \cdot \theta \sin 4\theta}{384} - \frac{k^4 \cdot \cos 4\theta}{384} (\ln \frac{2}{k} + 1,0895) \quad (2)$$

in which “ln” indicates the natural logarithm and

$$k = d \sqrt{\omega \mu / \rho_a} \quad d = \sqrt{(h_a + h_b)^2 + x_{ab}^2} \quad \theta = \tan^{-1} \frac{x_{ab}}{h_a + h_b}$$

It is noteworthy that when the angle θ is zero and $d = 2h_a$, the latter expressions for P_θ and Q_θ become the former ones used for the self impedance.

For values of $k > 5$ Carson gives another approximating series for P_θ and Q_θ :

$$P_\theta = \frac{\cos(\theta)}{\sqrt{2} \cdot k} - \frac{\cos(2\theta)}{k^2} + \frac{\cos(3\theta)}{\sqrt{2} \cdot k^3} + 3 \frac{\cos(5\theta)}{\sqrt{2} \cdot k^5} \quad (3)$$

$$Q_\theta = \frac{\cos(\theta)}{\sqrt{2} \cdot k} - \frac{\cos(3\theta)}{\sqrt{2} \cdot k^3} + 3 \frac{\cos(5\theta)}{\sqrt{2} \cdot k^5} \quad (4)$$

Further approximation and definition of an Equivalent Distance

As already stated, if the k coefficient is small enough, it is possible to neglect some terms in the Carson self- and mutual-impedance. The 0-degree approximation consists in neglecting the terms in k , except the logarithmic term in Q_θ , in the above equations. There is only one term left in P_θ and two in Q_θ :

$$P_\theta = \frac{\pi}{8} \quad (5)$$

and

$$Q_\theta = -0,03861 + \frac{1}{2} \ln \frac{2}{k} \quad (6)$$

For the mutual impedance this results in the following two simple expressions for R_{ba-g} and X_{ba-g} :

$$R_{ba-g} = 4 \cdot 10^{-7} \omega P_\theta = 10^{-7} \omega \frac{\pi}{2} = 10^{-7} f \pi^2 = 10^{-7} \frac{\pi^2}{T}$$

It is interesting to note that this resistance does not depend on soil resistivity. This paradox can be explained by the fact that when resistivity is higher, the current spreads over a larger area. For GIC currents whose period T may be supposed to range between 50 and 500 s [2], R_{ba-g} varies from $0,02 \cdot 10^{-6} \Omega/m$ to $0,002 \cdot 10^{-6} \Omega/m$ which is several orders of magnitude lower than the resistance of the line conductor(s).

$$X_{ba-g} = 2 \cdot 10^{-7} \omega \log\left(\frac{d}{D}\right) + 4 \cdot 10^{-7} \omega \left[-0,03861 + \frac{1}{2} \log\left(\frac{10^3}{1,405} \frac{1}{d} \sqrt{\frac{\rho_a}{f}}\right) \right]$$

After working out the second term by introducing the constant term into the logarithm and combining the resulting two logarithmic terms one finds that the physical distance d drops out of the equation:

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$$X_{ba-g} = 2 \cdot 10^{-7} \omega \ln \left(\frac{658,74 \sqrt{\frac{\rho_a}{f}}}{D} \right)$$

This expression of X_{ba-g} appears to have the same structure as the one given above for the mutual reactance when considering a perfectly conducting ground, but the numerator is no longer related to the physical distance d and may be considered as an Equivalent Distance D_e to a fictitious specular return under a perfectly conducting ground. It is sufficient to replace the distance d between conductor a (electrojet) and the image of conductor b by this Equivalent Distance D_e given by:

$$D_e = 658,74 \sqrt{\frac{\rho_a}{f}} \text{ m}$$

These very simple expressions or even the previous more elaborated Carson approximations for resistance and reactance are interesting since they are easily evaluated over a range of usual frequencies and resistivities with a spreadsheet, but are they valid to evaluate the GIC effects ?

Caveat

An important caveat must, indeed, be made as to the applicability of the approximating formulas for the correction terms given above in extreme situations as when studying the coupling of the electrojet with high voltage lines. The applicability is, as stated in Carson's original paper, restricted by the condition that $k < 1$ for the approximations (1) and (2) or larger than 5 for (3) and (4), and the restriction on the validity for the very simple expressions introducing the Equivalent Distance must obviously be even more severe.

As the purpose of this paper is evaluating the induced electric field in a HV line and the resulting magnetic induction at ground level, the electrojet is represented by a conductor at a height of 100 km carrying a current of 100 kA with a period of 360 s. Because the penetration depth at this low frequency reaches the deeper soil layers, the higher values of ground resistivity are considered, with particular emphasis on $\rho_a = 3333 \Omega\text{m}$ used in [2]. It is easily verified that the value of D_e under these circumstances appears to be some 721 km, which is much larger than the height of the electrojet and it seems to make sense to represent the return path by a conductor at this depth. For the more elaborated Carson formulas the value of k must be verified and, if the HV line is assumed to be parallel to and under the electrojet at 25 m height, the value of k is given by:

$$k = (100\,000 + 25) \sqrt{\frac{\omega \mu}{\rho_a}} = 0,2565$$

It appears thus that under these circumstances the value of k nearly violates the condition that k be less than 1. A verification of the evolution of these approximating equations as a function of the value of k is therefore on the order.

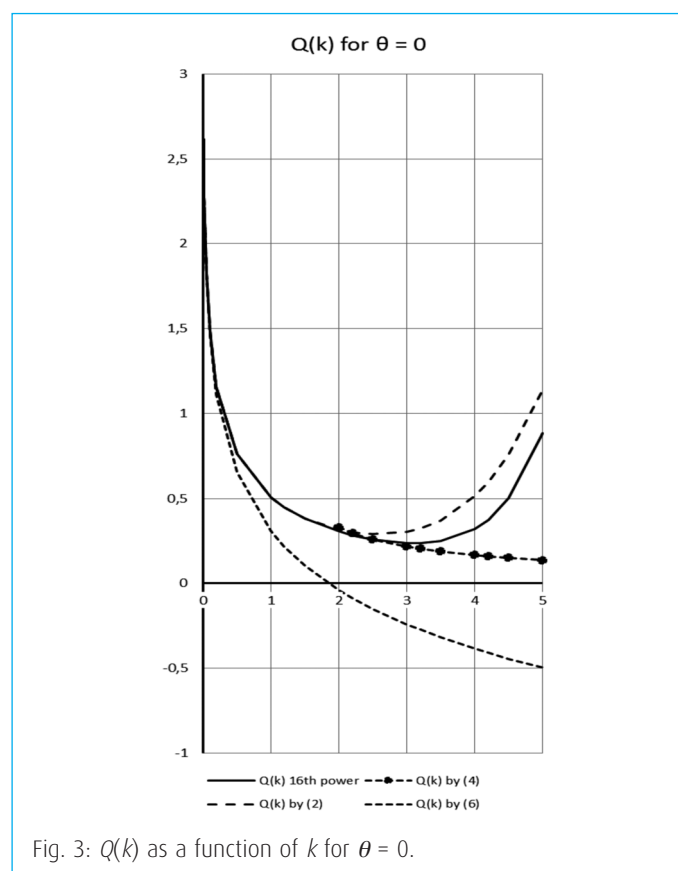


Fig. 3: $Q(k)$ as a function of k for $\theta = 0$.

A recursive inclusion of the full Carson series terms has been published in the literature [3] and has been used in industrial programs. Because of its recursive nature this series development includes as many terms in the series approximation as one may desire. For this paper the development of the series has been carried out to the 16th power of k . The Carson formula (2) is in fact that same development but carried out only up to the fourth power of k . Both the 16th power series and the formula (2) have been plotted together with the sole logarithmic term formula (6), which was the basis of the Equivalent Distance approach and with the large value approximation (4) in Fig. 3 for $\theta = 0$.

It may be observed that the series development up to 16th power of k is bending upwards for values $k > 3$ and its validity as a reference may therefore be questioned because from that same value of k on the formula (4) takes over and continues decreasing smoothly for higher values of k . As an equal trend appears in the evolution of P_0 , the zone $2 < k < 5$ may therefore be considered unreliable.

Therefore the industrial program EMTP relies today on another approach [4]. Instead of the evaluation of Carson's integrals for the correction terms, it uses a complex penetration depth in a direct formulation of the self and mutual impedances. Although they are still not an analytical integration of the original Carson integrals they achieve a remarkably accurate result when compared with numerical integration of the exact Carson equations. The deviations don't exceed a few percent over the whole range [5]. They certainly have the advantage that they reliably cover the range $2 < k < 5$.

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Using the Carson model with a spreadsheet

It is straightforward to implement the Carson equations mentioned above on a spreadsheet. With the help of simple dedicated macros, it is even possible to quickly study the influence of parameter variation, but the verification of the validity is mandatory in order to avoid the unreliable range, where the use of the complex penetration depth would be necessary. The implementation of complex penetration depth formulas is, however, much more cumbersome in spreadsheets, but EMTP relies on its use and may therefore be used to check the accuracy for different approximations.

As an example, Table 1 makes this comparison for an electrojet of 100 kA and period equal to 360 s at 100 km height with a ground resistivity of 3333,33 Ωm or a conductivity of 0,0003 S/m, k being then 0,2565 as shown above.

In order to simulate the influence of a sheet of current rather than a filament it is possible to raise the height of the electrojet and increase proportionally the current to maintain the B_h at the given value of 222,4 nT as explained hereafter in the EMTP paragraphs. For 5000 km height, the k coefficient is equal to 12,825. The polynomial developments, even to the 16th power of k , are no longer valid as shown in Fig. 3. The other series (3) and (4) given above have to be used and k is surely outside of the unreliable zone. For $B = 222,4$ nT, we obtain $E = 1,512$ V/km, exactly the value given by EMTP as shown hereafter, and also in [2].

Using the Electrojet Model with EMTP

Electrojet Model implemented in EMTP

The present chapter presents the results of the GIC calculation approach, using the LINE CONSTANTS routine of EMTP. We refer to [3], [4] and [5] for the implementation of ground return influence in the routine LINE CONSTANTS of EMTP.

The output of LINE CONSTANTS is a susceptance matrix as well as an impedance matrix linking all conductors introduced in the calculation. These matrices are normally used in the specific EMTP transient calculations performed with the main module of EMTP.

For the phenomenon of GIC (extremely low frequency), the susceptance matrix is of no concern.

We further concentrate on a practical example for GIC and the corresponding impedance matrix: the single conductor model for an electro-jet at 100 km altitude and 3 conductors near earth as per Fig. 4.

EMTP printed output

The following data is entered in the original 80 columns EMTP input format:

- ELECTROJET = Conductor (01) at 100.000 m height;
- three conductors (02,03 and 04) near earth at 0,10 m, 25 m and 35 m height;

Table 1: Comparison of B_h at ground level and induced voltage E for an electrojet of 100 kA

| Power of k | B_h (nT) | Error vs EMTP | E (V/km) | Error vs EMTP |
|--------------|------------|---------------|------------|---------------|
| 0 | 200,4 | – 9,9 % | 0,742 | – 3,4 % |
| 1 | 225,9 | 1,6 % | 0,768 | 0,0 % |
| 4 | 222,9 | 0,2 % | 0,769 | 0,1 % |
| 16 | 222,5 | 0,0 % | 0,769 | 0,1 % |
| EMTP | 222,4 | | 0,768 | |

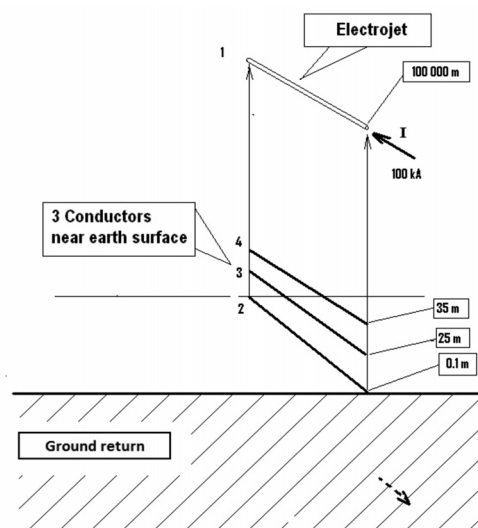


Fig. 4: Model of electrojet at 100 km height and 3 conductors near earth surface

Impedance matrix, in units of [ohms/kmeter] for the system of physical conductors. Rows and columns proceed in the same order as the sorted input.

| | 1 | 2 | 3 | 4 |
|---|------------------------------|------------------------------|------------------------------|------------------------------|
| 1 | 2.147317E-06 4.314064E-05 | | | |
| 2 | 2.401330E-06 7.304956E-06 | 5.220274E-02 6.246454E-05 | | |
| 3 | 2.401258E-06 7.305921E-06 | 2.743649E-06 3.589234E-05 | 5.220274E-02 6.246475E-05 | |
| 4 | 2.401229E-06 7.306308E-06 | 2.743607E-06 3.471292E-05 | 2.743502E-06 3.907951E-05 | 5.220274E-02 6.246483E-05 |

Fig. 5: Impedance matrix in output format of EMTP

- RHO earth ρ_a 3333 Ωm ;
- frequency 0,00278 Hz of electrojet waveshape (corresponds to a cycle period of 360 s).

The impedance matrix produced by EMTP as output is shown in Fig. 5. A few observations are of interest:

- mutual impedances between Electrojet (cond 1) and the three conductors near earth (cond 2, 3, 4) are practically identical $R_{mut} = 2,401 \cdot 10^{-6} \Omega/\text{km}$ and $X_{mut} = 7,305 \cdot 10^{-6} \Omega/\text{km}$; this implies that the GIC induces almost the same voltage in all conductors running parallel near earth surface;
- the Self impedances of the conductors near earth are almost purely resistive (ratio $R/X \sim 10^3$), with R practically equal to the resistance of the conductor proper; this gives confirmation of the simplifying assumption in the 'Electromagnetic' approach that fields associated with currents flowing as a consequence of geomagnetic induced voltages can be neglected;

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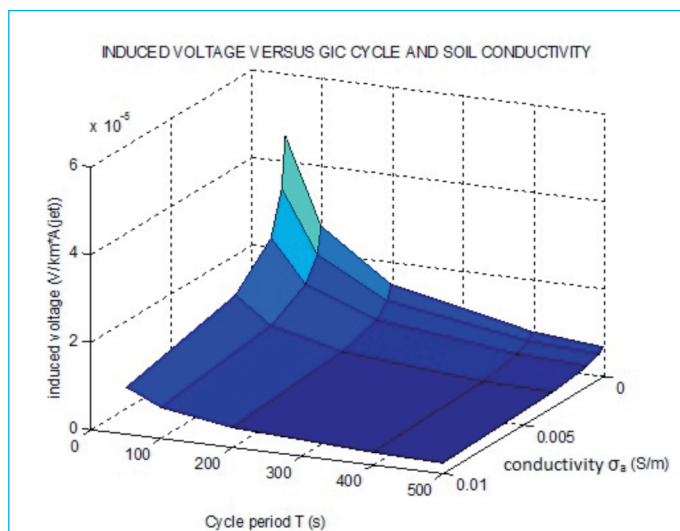


Fig. 6: Sensitivity of induced voltage as function of the cycle period T and soil conductivity σ_a .

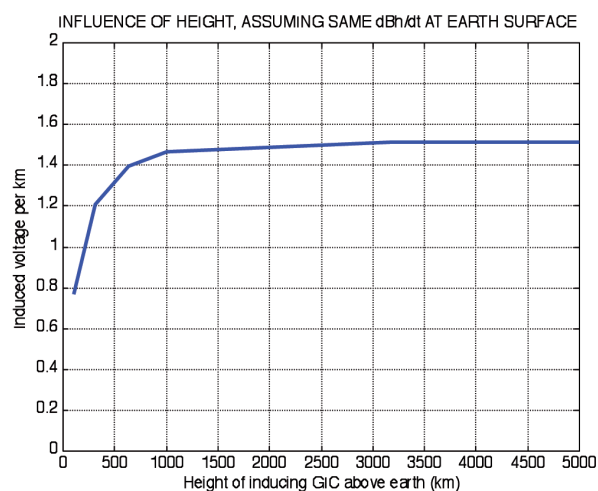


Fig. 7: Induced voltage per km as function of the height of electrojet

– mutually induced voltages (assuming even induced currents up to 1000 A) between the conductors near earth are negligible compared to the GIC induced voltages; using the greater values of the Z matrix above:

$$10^5 A^* 7,306 \cdot 10^{-6} \Omega/\text{km} \gg \gg \sim 10^3 A^* 3,908 \cdot 10^{-5} \Omega/\text{km} \\ 0,7306 \text{ V/km} \gg \gg \sim 0,03908 \text{ V/km}$$

Relation between Electrojet Model and variation of B_h at ground level

The actual current flowing in the Electrojet is not directly observable, but it causes a variation of magnetic field at ground level. This latter phenomenon is easily observable and monitored by measuring stations worldwide. In order to be practical, the Electrojet Model needs a link with this observed induction.

Although a circuit model is used, it provides an easy link with the induction at ground level when considering simultaneously two conductors vertically superposed near the ground level (e.g. conductors 2 and 3 in Fig. 4).

The voltage induced in the loop formed by conductors 2 and 3 (Fig. 4) can be considered as:

- The difference of the voltage induced by the Electrojet in the upper conductor minus the voltage induced by the same in the lower conductor evaluated from their respective impedance matrices;
- The voltage created by Faraday-Lenz's law due to the variation of the perpendicular B component through the surface delimited by the upper and lower conductor. The induction at ground level is obtained by equivalencing these two expressions for the induced voltage.

Applying this to the sample case (with input data and output values given above) we find:

- Induced voltage in the loop formed by conductor 2 (0.1 m above ground) and 3 (25 m above ground) = $|1,0 \cdot 10^5 \cdot ((2,401330 \cdot 10^{-6} + j7,304956 \cdot 10^{-6}) - (2,401258 \cdot 10^{-6} + j7,305921 \cdot 10^{-6}))| \text{ V/km}$
- Faraday-Lenz's induced voltage in the same loop (per km length of loop) = $|B_h| \cdot (2 \pi \cdot 2,780 \cdot 10^{-3}) \cdot (25,0 - 0,1) \cdot 1000 \text{ V/km}$
- Equalling both expressions results in $|B_h| = 2,2249 \cdot 10^{-7} \text{ T}$ or 222,49 nT
- Using the same approach with conductor 2 and 4 (35m above ground) results in $|B_h| = 2,2240 \cdot 10^{-7} \text{ T}$ or 222,40 nT
- We may conclude that with the conditions as given in the sample case (earth resistivity, cycle time of Electrojet, 100 kA Electrojet at 100 km altitude) the amplitude of the horizontal magnetic induction variation is 222,4 nT, and the induced voltage in each of the conductors is nearly:
 $|1,0 \cdot 10^5 \cdot (2,401330 \cdot 10^{-6} + j7,304956 \cdot 10^{-6})| = 0.769 \text{ V/km}$

This holds for filament type electrojet. The case of flat current sheet approximation is covered in the application 2 hereafter.

Application 1: Induced voltage versus soil conductance and GIC cycle period

The first application shows a 3D-graph (Fig. 6) of induced voltage against parameters T and σ_a for an Electrojet of 100 kA at 100 km altitude with:

- soil conductivity σ_a range: $0,0001 < \sigma_a < 0,01 \text{ S/m}$;
- GIC cycle period T range: $50 < T < 500 \text{ s}$.

This graph may be compared with information reproduced in the companion paper [2], and it shows a good approximation of values found with other calculation methods.

Application 2: Approximation of a sheet conductor Electrojet by increased height of a filament Electrojet

As explained in detail in the companion paper [2], sheet type electrojets create flat waves (with only Z-coordinate dependency), and these induce (for the same dB_h/dt near ground) higher voltages on near-ground conductors than filament type electrojets.

The higher the altitude of filament electrojet conductor (with correspondingly increasing GIC current, so as to

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maintain dB_H/dt constant), the more the field approximates the 'only Z-coordinate dependency'.

The following Fig. 7 illustrates this case with constant dB_H/dt values, for electrojet filament altitudes ranging from 100 km to 5000 km (and consequently currents ranging from 100 kA to approximately 5000 kA) reaching an asymptotical value of 1,51 V/km, a value corresponding to the value for a sheet of current cited in the companion paper [2] and to the value obtained by the spreadsheet approximation cited earlier in this paper.

A third application deals with the spreading of a filament electrojet into a number of filaments at 20 km distance of each other at the same altitude. An equal current in all conductors is adjusted so as to keep the induction under the middle conductor constant. As the number is increased (3 to 7 to 11) the field tends also towards the one-dimensional flat field, but the increase of induced voltage observed (~ 20 %) makes us conclude that even a flat GIC strip of width = $2 \times$ height above earth is far away from the theoretical flat case which achieves an increase of induced voltage of nearly 200 % (see ref [2] and the graph in Fig. 7). The flat case is likely to be approximated only by using closer filaments and a sheath width well above $10 \times$ the sheath height above earth.

Conclusion

This paper has shown that 1) Carson formulas may be used in a spreadsheet to obtain the induced electric fields in the soil for a given electrojet current and the precision obtained is satisfactory depending on the value of the variable k ; that 2) it is possible to link the magnitude of the electrojet model, be it a filament current or a sheet current, to the measured horizontal magnetic induction at ground level; that 3) within the validity range excellent agreement of results is obtained by spreadsheet calculations with those obtained by an industrial EMT program and with those obtained by a more dedicated scientific approach in a companion paper [2].

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